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A logistics network design model with vendor managed inventory

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ABSTRACT

In this paper, we study a logistics network design problem with vendor managed inventory in which the company is in charge of managing inventory for its downstream warehouses and retailers, and can choose whether to satisfy each retailer's demand. The problem incorporates the location, transportation, pricing, and warehouse-retailer echelon inventory replenishment decisions. Traditionally, these decisions are made separately. We formulate the problem as a set-packing model and solve it using branch-and-price. The pricing problem that arises from each iteration of the column generation procedure is an interesting nonlinear IP problem. We show the pricing problem can be solved in $O(n^2 \log n)$ time for each warehouse, where n is the number of retailers. The computational results shed insights on the benefits that the integrated approach can achieve significant profit improvement. The computational results also highlight the efficiency of the solution algorithm.

1. Introduction

Vendor managed inventory (VMI) is an effective supply chain planning technique that aims at reducing logistics cost and improving service by coordinating the operations of different logistical entities across the supply chain. Traditionally, each logistical entity involved in the supply chain manages its own inventory independently. By centralizing the inventory control and coordinating the multi-echelon inventory replenishment under VMI, the system-wide logistics cost can be significantly reduced and the service level can be improved. As shown by Simchi-Levi et al. (2003), Ballou (2004), and Yang et al. (2010), a unified systems approach is required to successfully implement VMI, which can help effectively integrate the supplier, its downstream warehouses and retailers so that the product is produced and distributed at the right quantities, to the right locations, and at the right time. Motivated by this recent popular supply chain initiative — vendor managed inventory, in this paper, we study a logistics network design problem integrating multi-echelon inventory management under the VMI framework in which the supplier manages the inventory of a single product for its downstream warehouses and retailers. Under this VMI framework, the system-wide inventory, including the inventory maintained at both warehouses and retailers, is owned by the supplier. The supplier is the sole decision maker who is in charge of the warehouse-retailer echelon inventory replenishment, the transportation of the product from it to the warehouses and from the warehouses to the retailers, and the price of the product. The warehouse-retailer echelon inventory is owned by the supplier until it is sold. The goal of the supplier is to maximize the total profit.

A supply chain distribution network's physical structure can substantially affect its performance and profit margin. Most existing research on supply chain network design pursues a cost-minimization objective and tries to satisfy all the demands. However, the additional revenue generated from serving some retailers could be much lower than the cost associated with serving them. Thus, trying to satisfy all the retailers' demands might not give us the highest profit. As shown by Shen (2006), it could be more profitable for a company to lose some potential demands to competitors. It is difficult to determine whether it is profitable to serve each individual retailer a priori. Thus, we intend to propose a logistics network design model that can help simultaneously determine the location, warehouse-retailer assignment, warehouse-retailer echelon inventory replenishment, sale price, and which set of retailers to serve. The problem can be described as follows. We consider a company that produces a single product in a production site. We are given a set of retailers each of which faces a deterministic demand at a constant rate. We are also given a set of potential warehouse locations and each warehouse is assumed to be uncapacitated. The product will be shipped from the production site to certain retailers via some selected warehouses. The company wants to determine (i) the number and locations of the warehouses to open and the retailers

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to serve, (ii) the retailer assignments, (iii) the warehouse-retailer echelon inventory replenishment policy, and (iv) the sale price of the product associated with each warehouse located, so as to maximize the total profit which equals to the total revenue minus the total cost. The cost components include the warehouse establishing and operating cost, the warehouse-retailer echelon inventory related cost, and the shipment cost from the production site to the warehouses open and from each open warehouse to respective retailers served. The cost of establishing and operating a warehouse is fixed, which is assumed to be independent of the number of retailers assigned to that warehouse.

Comparing with other logistics network design models in the literature, the novelties of our model lie in the following three aspects. First, it brings the VMI concept into logistics distribution network design with a profit-maximizing objective. Second, it allows the supplier to decide whether to serve each retailer, i.e., the supplier can choose the set of retailers to serve within the multi-echelon inventory management context. This gives rise to a set-packing profit-maximizing model. It is unlike other traditional multi-echelon logistics network design models which pursue a cost-minimizing objective and try to satisfy all the demands. Third, we use extensive computational experiments to demonstrate the potential practical impact of integrated decision-making by comparing the solutions obtained from our model with the traditional sequential decision-making process. The average benefit ranges from 10.4% to 21.7% in terms of the total profit. The computational results also shed insights on the benefits of our model with the supplier having retailer-serving flexibility over the traditional model that requires all the demands should be served.

The rest of this paper is organized as follows. In Section 2, we review the related literature. In Section 3, we present a set-packing model with a profit-maximizing objective for our network design problem. In Section 4, we study the solution procedure which includes the solution to the pricing problem and a speed up heuristic for column generation. In Section 5, we present a traditional sequential decision-making approach for the problem. In Section 6, we report and discuss the computational results. Finally, we outline a few generalizations of our model and conclude the paper in Sections 7 and 8, respectively.

2. Literature review

The integrated logistics network design optimization received increasing attentions in the literature recently. One important stream of this research focuses on the single-echelon risk-pooling network design problems with a single supplier and with warehouses serving as the intermediate facilities between the supplier and the retailers, and holding two types of inventory: the working inventory and the safety stock. Shen et al. (2003) formulate the uncapacitated risk-pooling network design problem as a set-covering model and solve it using a column generation approach when the mean-to-variance ratio of the demand at each retailer is identical for all retailers. Daskin et al. (2002) solve the same problem using a Lagrangian relaxation based approach. Shu et al. (2005) propose an efficient algorithm for the subproblem by relaxing the assumption on the mean-to-variance ratio being identical for all retailers and solve the problem using column generation. Qi et al. (2010) study a similar problem with supply disruptions. Miranda and Garrido (2006, 2009) and Ozsen et al. (2008, 2009) study various capacitated risk-pooling network design models. Sourirajan et al. (2007, 2009) study a more general risk-pooling network design problem in which the replenishment lead time is explicitly modeled. They develop a Lagrangian relaxation based heuristic and a genetic algorithm, respectively, to solve the problem. Park et al. (2010) study a risk-pooling network design problem with the consideration of selecting suppliers and lead times being DC-to-supplier dependent. Shen (2005), Snyder et al. (2007), and Vidyarthi et al. (2007) study multi-commodity risk-pooling network design problems. While the last one considers the selection of suppliers and ignores the working inventory in the system. Manzini and Bindi (2009) and Gebennini et al. (2009) continue this line of research by further considering the production decisions. Shen and Qi (2007) and Javid and Azad (2010) study the single-echelon risk-pooling network design problems integrating routing costs. Some other important single-echelon logistics network design models are contributed by Candas and Kutanoglu (2007) and Jeet et al. (2009) for the service parts industry and Lee et al. (2010) for the sustainable logistics network design.

In another stream of this research, the multi-echelon inventory replenishment related cost is considered. Teo and Shu (2004) propose a two-echelon location-inventory distribution network design problem in which both warehouses and retailers carry inventory and each retailer faces a deterministic demand at a constant rate. They solve the problem using column generation. Üster et al. (2008) study a similar problem in which they assume a single warehouse is located and the location decision is continuous, and propose several heuristic algorithms that are both effective and efficient to tackle it. Keskin et al. (2010) study a stochastic two-echelon location-inventory distribution network design problem.

The literature on retailer demand selection and profit maximization in logistics is also related to our problem, for example, Bakal et al. (2008), Chahar and Taaffe (2009), Geunes et al. (2004, 2005, 2006, 2011), and Taaffe et al. (2008a,b).

Nevertheless, all the models proposed and studied by the aforementioned papers either do cost minimization for network design or ignore the location decision. The literature on profit-maximizing supply chain network design is rather limited. Zhang (2001) studies a profit-maximizing location model in which a single warehouse is located and the price for its product is the same. It assumes that if the price charged by the supplier is higher than a customer's reserve price, the supplier will lose this customer. The model does not consider any cost terms. In a recent paper, Shen (2006) presents a profit-maximizing supply chain design model in which the location, transportation, and inventory replenishment related costs at the warehouse level are considered.

3. Model development

In this section, we first define the notations, assumptions, and then develop a set-packing model for the logistics network design problem with vendor managed inventory.

3.1. Notations and assumptions

To model the logistics network design problem with vendor managed inventory we first define the following notations: **Sets**

I	set of the retailers, $\lvert I\rvert = n$;
W	set of the potential warehouse locations, $\lvert W\rvert = m$.

Inputs and parameters

λ_i	annual demand rate faced by retailer i for each $i \in I$;
$K_{w,0}$	fixed ordering cost incurred by warehouse w every time it places an order to the supplier for each $w \in W$. It is independent of the ordering quantity;

K_i fixed ordering cost incurred by retailer i every time it places an order to the warehouse assigned to it for each $i \in I$. It is independent of the ordering quantity;
$h_{w,0}$ inventory holding cost rate at warehouse w per unit of the product per year for each $w \in W$;
h_i inventory holding cost rate per unit of the product per year at retailer i for each $i \in I$;
$d_{w,0}$ distance from the supplier to warehouse w for $w \in W$;
$d_{i,w}$ distance from warehouse w to retailer i for $i \in I$, $w \in W$;
$c(d_{w,0})$ transportation cost per unit of the product shipped from the supplier to warehouse w for $w \in W$. It is assumed to be a non-decreasing stepwise function with the increase of $d_{w,0}$;
$g(d_{i,w})$ transportation cost per unit of the product charged to retailer i if the shipment is made from warehouse w. It is assumed to be a non-decreasing stepwise function with the increase of $d_{i,w}$;
F_w annual fixed cost of operating warehouse w per year for each $w \in W$;
v_i reserve price of retailer i for each $i \in I$.

Decision variables

$T_{w,0}$ inventory reorder interval at warehouse w for each $w \in W$;
T_i inventory reorder interval at retailer i for each $i \in I$;
p_w sale price per unit of the product at the set of retailers served by warehouse w for each $w \in W$;
$x_{w,S}$ binary variable which equals to 1 if warehouse w is used to serve retailers in S and no one else; and 0, otherwise.

Under the vendor managed inventory assumption, the company owns the inventory carried in both the warehouses and the retailers until it is sold. The company is in charge of managing the inventory for the warehouses and the retailers served and designing the corresponding inventory replenishment policy. We also assume that there is a sale price associated with the retailers assigned to each warehouse open. Due to the existence of competition, we assume each retailer holds a reserve price for the product. We further assume that if the total price of warehouse w (which equals to the sale price of warehouse w plus the shipment cost from warehouse w to retailer i) charged to retailer i is higher than retailer i's reserve price, i.e., $p_w+g(d_{i,w})>v_i$, then the company will lose retailer i. Thus, we can define (cf. Shen, 2006; Zhang, 2001):

$$r_i(p_w,d_{i,w}) = \begin{cases} p_w+g(d_{i,w})-c(d_{w,0}), & p_w+g(d_{i,w}) \le v_i, \\ 0, & p_w+g(d_{i,w}) > v_i. \end{cases}$$

3.2. Model formulation

With the notations and assumptions summarized in the previous section, the logistics network design problem with vendor managed inventory can be modeled as a set-packing model as follows:

$$\begin{aligned} P:\ \max\ & \sum_{w \in W}\sum_{S \subseteq I} R_{w,S}x_{w,S} \\ \text{s.t.}\ & \sum_{w \in W}\sum_{S \subseteq I: i \in S} x_{w,S} \le 1, \quad \forall\, i \in I, \\ & x_{w,S} \in \{0,1\}, \quad \forall\, w \in W,\ S \subseteq I, \end{aligned}$$

where $R_{w,S}$ denotes the profit of serving retailers in S using warehouse w for $w \in W$ and $S \subseteq I$. We assume if the supplier decides to serve a retailer, then it must satisfy all this retailer's demand from a warehouse. On the contrary, if a retailer is not served by the supplier, then the supplier will not serve any of this retailer's demand and thus, gain no revenue from it. $R_{w,S} = \sum_{i \in S} r_i(p_w,d_{i,w})\lambda_i - \bar{I}(w,S) - F_w$, where $\bar{I}(w,S)$ denotes the optimal warehouse-retailer echelon inventory cost. According to a seminal work by Roundy (1985), the cost term $\bar{I}(w,S)$ can be approximated within 98% accuracy by

$$I(w,S) = \min_{T_{w,0},T_i>0, i \in S} \left(\frac{K_{w,0}}{T_{w,0}} + \sum_{i \in S}\frac{K_i}{T_i} + \frac{1}{2}\sum_{i \in S}\lambda_i h_i T_i + \frac{1}{2}\sum_{i \in S}\lambda_i h_{w,0}[\max(T_i,T_{w,0})-T_i] \right),$$

i.e., $I(w,S) \le \bar{I}(w,S) \le 1.02I(w,S)$. In the rest of this paper, we will use $I(w,S)$ to approximate the optimum two-echelon inventory cost function $\bar{I}(w,S)$. Thus, we redefine $R_{w,S} \equiv \sum_{i \in S} r_i(p_w,d_{i,w})\ \lambda_i - I(w,S) - F_w$.

We note that the solution to our problem is a maximum profit partition of the subset of retailers I into disjoint sets $(S_1,\dots,S_k)$ (i.e., $S_1 \cup S_2 \cup \cdots \cup S_k \subseteq I$ and $S_i \cap S_j = \emptyset, \forall i \ne j$) together with the corresponding warehouse assignment $(w_1,\dots,w_k)$. Following the constraints in the set-packing model, we assume each retailer is served by at most one open warehouse, i.e., if a retailer is served, then the single-sourcing requirement is enforced.

This model is clearly NP-hard and contains exponentially many of variables $(|W|2^{|I|})$. We can apply a branch-and-price approach to solve this integer programming problem. Branch-and-price is commonly used to solve network design problems to optimality (cf. Andersen et al., 2011). In implementing branch-and-price to it, column generation is used to solve its linear programming relaxation in each iteration of the branch-and-bound procedure. We start each iteration by solving the linear programming relaxation of (P) with a subset of columns which is called the restricted master problem. The initial set of columns should include all singletons. For each column (w,S), we want to know whether its reduced cost is non-positive. If the answer is yes, then the solution of this iteration is optimal to the LP relaxation of (P). Otherwise, a column (w,S) with a positive reduced cost is found; then we add this column to the restricted master problem solved in the last iteration and start the next iteration.

4. Solution procedure

We use column generation embedded in a branch-and-bound procedure to solve problem (P). In this section, we show how to efficiently solve the pricing problem, and derive lower and upper bounds of problem (P). We also present a variable fixing technique to speed up the algorithm.

4.1. The pricing problem

Let $\{u_i, i=1,2,\dots,|I|\}$ be the optimal dual solution obtained in one of the iterations of the column generation approach to the linear programming relaxation of (P). For each column (w,S), we want to know whether the reduced cost $R_{w,S}-\sum_{i \in S}u_i$ for each $w \in W$ and $S \subseteq I$ is non-positive. For a fixed w, this is equivalent to checking whether

$$P_w: \max_{S \subseteq I}\sum_{i \in S} r_i(p_w,d_{i,w})\lambda_i - I(w,S) - F_w - \sum_{i \in S}u_i > 0,$$

which we call the pricing problem. We note that the above pricing problem is significantly different from the one studied in Shen (2006) due to the multi-echelon inventory cost function involved. The nonlinear multi-echelon inventory cost component $I(w,S)$ features the dominant difference in our pricing problem.

Clearly, (P_w) is equivalent to

$$\min_{S\subseteq I} I(w,S)+\sum_{i\in S}u_i-\sum_{i\in S}r_i(p_w,d_{i,w})\lambda_i \le -F_w.$$

Let $a_i(p_w)\equiv r_i(p_w,d_{i,w})\lambda_i-u_i$. Thus, in order to check whether (P_w) is non-positive, we need to solve

$$\min_{S\subseteq I} I(w,S)-\sum_{i\in S}a_i(p_w). \tag{1}$$

For a fixed p_w, (1) can be solved in $O(n\log n)$ steps as shown in Teo and Shu (2004). Let S' be the optimal solution to (1) for some fixed p_w, and $T^*_{w,0}(S',p_w)$ and $T^*_i(S',p_w)$ (abbreviated by $T^*_{w,0}$ and T^*_i, respectively) be the corresponding optimal replenishment interval at warehouse w and retailer i, respectively. In the solution to (1) with a fixed p_w, the retailers in S' can be classified into three groups by the convexity of (1) and the KKT conditions:

- $i\in L_w=\{i: T^*_i=\sqrt{2K_i/\lambda_i(h_i-h_{w,0})}<T^*_{w,0}\}$ if and only if

$$\sqrt{2K_i\lambda_i(h_i-h_{w,0})}+\frac{1}{2}\lambda_i h_{w,0}T^*_{w,0}-a_i(p_w)<0; \tag{2}$$

- $i\in E_w=\{i: T^*_i=T^*_{w,0}=\sqrt{2[K_{w,0}+\sum_{i\in E_w}K_i]/[\sum_{i\in E_w}\lambda_i h_i+\sum_{i\in L_w}\lambda_i h_{w,0}]}\}$ if and only if

$$\frac{K_i}{T^*_{w,0}}+\frac{1}{2}\lambda_i h_i T^*_{w,0}-a_i(p_w)<0; \tag{3}$$

- $i\in G_w=\{i: T^*_i=\sqrt{2K_i/\lambda_i h_i}>T^*_{w,0}\}$ if and only if

$$\sqrt{2K_i\lambda_i h_i}-a_i(p_w)<0; \tag{4}$$

where $G_w\cup L_w\cup E_w=S'\subseteq I$.

Teo and Shu (2004) show that in order to use (2)–(4) to check the membership and obtain S', we can partition a real line into a collection of intervals for $T^*_{w,0}$. Each interval along this line, say $[a,b]$, should satisfy the following properties:

- The open interval (a,b) does not contain any of the points $b^1_i\equiv\sqrt{2K_i/\lambda_i(h_i-h_{w,0})}$ and $b^2_i\equiv\sqrt{2K_i/\lambda_i h_i}$ for $i=1,\dots,n$.
- The open interval (a,b) does not contain the roots to the following linear and quadratic equations in terms of $T^*_{w,0}$:

$$\sqrt{2K_i\lambda_i(h_i-h_{w,0})}+\frac{1}{2}\lambda_i h_{w,0}T^*_{w,0}-a_i(p_w)=0, \tag{5}$$

to which the roots are given by

$$c_i(p_w)\equiv\frac{2a_i(p_w)-2\sqrt{2K_i\lambda_i(h_i-h_{w,0})}}{\lambda_i h_{w,0}}$$

and

$$\frac{K_i}{T^*_{w,0}}+\frac{1}{2}\lambda_i h_i T^*_{w,0}-a_i(p_w)=0 \tag{6}$$

to which the roots are given by

$$d^{1,2}_i(p_w)\equiv\frac{a_i(p_w)\pm\sqrt{(a_i(p_w))^2-2\lambda_i h_i K_i}}{\lambda_i h_i},$$

for $i=1,\dots,n$.

Thus, for a fixed p_w, (1) can be solved by sorting the positive values of $(b^1_i,b^2_i,c_i(p_w),d^{1,2}_i(p_w))$ for all $i\in I$ and use these values to partition a real line into a collection of intervals.

However, p_w is also a decision variable in the pricing problem. Thus,it will not be possible to implement the above procedure for all values of p_w to solve (1) since even a small change of the value of p_w might cause the change of the ordering of $(b^1_i,b^2_i,c_i(p_w),d^{1,2}_i(p_w))$. To avoid this,we need to partition another real line (e.g., $G=[0,g]$) into a collection of small intervals for p_w. For p_w taking any value within each interval,say $[c,d]$,we need to guarantee that the ordering of $(b^1_i,b^2_i,c_i(p_w),d^{1,2}_i(p_w))$ does not change. We note that as long as the ordering of $(b^1_i,b^2_i,c_i(p_w),d^{1,2}_i(p_w))$ does not change, S' will not change and the sets of G_w,L_w, and E_w,and $T^*_{w,0}$ can be uniquely determined. In order to achieve this,each such interval,say $[c,d]$,in the collection must satisfy the following properties:

- The open interval (c,d) does not contain any of the points $f_i\equiv\sup\{p_w\in P \mid r_i(p_w,d_{i,w})>0\}$ for $i=1,\dots,n$.
- The open interval (c,d) does not contain the roots to the equations $\sqrt{2K_i\lambda_i h_i}-a_i(p_w)=0$,where we denote the roots to them by e_i for $i=1,\dots,n$.
- The open interval (c,d) does not contain the roots to the equations:
  - $b^1_i=c_j(p_w)$, $b^1_i=d^{1,2}_j(p_w)$, $\forall i,j\in I$;
  - $b^2_i=c_j(p_w)$, $b^2_i=d^{1,2}_j(p_w)$, $\forall i,j\in I$;
  - $e_i=c_j(p_w)$, $e_i=d^{1,2}_j(p_w)$, $\forall i,j\in I$;
  - $f_i=c_j(p_w)$, $f_i=d^{1,2}_j(p_w)$, $\forall i,j\in I$;
  - $c_i(p_w)=d^{1,2}_j(p_w)$, $d^1_j(p_w)=d^2_j(p_w)$, $\forall i,j\in I$;
  - $c_i(p_w)=c_j(p_w)$, $d^{1,2}_i(p_w)=d^{1,2}_j(p_w)$, $\forall i,j\in I, i\ne j$.

Let r_{ij} denote the roots to the above equations. We solve the above equations and ignore those r_{ij} whose values are non-positive since p_w should be positive. Then, we can partition the real line according to the positive values of r_{ij}, e_i, and f_i. For ease of exposition, we relabelled the points r_{ij},e_i, and f_i as r_k so that $r_k\le r_{k+1}$ for all $k=1,2,\dots,B$ where $B+1$ denotes the number of different values of r_{ij},e_i,f_i, and g.

Proposition 1. *The computational complexity of the pricing problem is $O(n^2\log n)$.*

Proof. From the construction above, it is clear that $B=O(n^2)$. Thus, implementing the sorting procedure for p_w can be executed in $O(B\log B)=O(n^2\log n)$ time. To summarize, after the execution of the sorting procedure, we obtain points r_k, $k=1,2,\dots,B$, with $0\le r_1\le r_2\le\cdots\le r_{B+1}=g$ (for guessing p^*_w which denotes the optimal p_w to the pricing problem) and for each interval $[r_k,r_{k+1}]$, we can use (2)–(4) to obtain the corresponding S'. Finally, the pricing problem (P_w) is solved by selecting the one with the minimum objective value.

We have shown that sorting the values of r_k requires $O(n^2\log n)$ time. For p_w being in each interval, the computational complexity for solving (P_w) is $O(n\log n)$ (cf. Teo and Shu, 2004). Totally, there are $O(B)=O(n^2)$ intervals for searching for p^*_w. Instead of searching all $O(n^2)$ intervals, we can implement a bisectional search procedure which only need to search at most $O(\log B)=O(\log n)$ intervals for p_w. Then, the total complexity for searching among these intervals is $O(n(\log n)\times(\log n))=O(n(\log n)^2)$, which is dominated by the sorting complexity of $\{r_k,k=1,2,\dots,B\}$ which is shown to be $O(n^2\log n)$. Thus, the pricing problem can be solved in $O(n^2\log n)$ steps. □

4.2. Speed up the algorithm

In the straightforward implementation of the column generation algorithm, we need to solve the pricing problem (P_w) for each $w\in W$. This usually results in a slow convergence of column generation. If we can know in advance that some warehouses will not open in the optimal solution, then we do not need to solve the pricing problems associated with these warehouses and those columns associated with these warehouses can also be removed from the column generation master LP problem. This will help to reduce the solution time significantly. In order to achieve this, we show how to construct the upper bound and the lower bound based on Lagrangian relaxation, and the information of the primal and dual solutions to determine whether a warehouse will open in an optimal solution early in the column generation procedure. Let Z_{IP} and Z_{LP} denote the optimal solution to the set-packing problem and its LP relaxation, respectively. At each iteration of the column generation procedure, we have a set of dual solutions $\{u_i : i \in I\}$, a set of feasible LP solution $x_{w,S}$, and the reduced cost $r_w \equiv \max_{S \subseteq I}(R_{w,S} - \sum_{i \in S} u_i)$.

4.2.1. Constructing an upper bound

The Lagrangian dual of the LP relaxation of (*P*) is

$$L(\lambda) = \sum_i \lambda_i + \max\left\{ \left(\sum_w \sum_S \left(R_{w,S} - \sum_{k \in S} \lambda_k \right) x_{w,S} \right) : 1 \geq x_{w,S} \geq 0, \forall\ w \right\}.$$

Therefore, $Z_{IP} \leq \min_\lambda L(\lambda) \leq L(u) = \sum_i u_i + \sum_{w: r_w \geq 0} r_w$, which gives rise to an upper bound.

4.2.2. Constructing a lower bound

The speed up technique depends largely on the quality of the lower bound (denoted as *LB*) of Z_{IP}. We generate a lower bound by constructing a feasible IP solution using the following heuristic:

Step 0: Let x^* be the optimal LP solution obtained by solving the problem using a partial set of columns. If x^* is integral, then the solution $\sum_{w,S} R_{w,S} x^*_{w,S}$ is a lower bound to Z_{IP}. Otherwise, we go to the next step.

Step 1: Order the retailers in a list according to the non-decreasing value of the demand.

Step 2: Start from the first retailer (say *i*) in the list. Do while the list is non-empty. If for some *S* and *w*, $i \in S$ and $x^*_{w,S} = 1$, then retailer *i* is served by warehouse *w*. Otherwise, there exists *S* and *T*, both of which contain *i*, and *w* and *w'*, such that $x^*_{w,S} > 0$ and $x^*_{w',T} > 0$. We serve *i* using the warehouse that will lead to the most total profit, and remove retailer *i* from the list.

In this way, we can generate a feasible IP solution to the set-packing model. We can use this solution as a bound to perform variable fixing as follows. Let w^* be a warehouse such that $r_{w^*} < 0$. If $\sum_i u_i + \sum_{w: r_w \geq 0} r_w + r_{w^*} < LB$, then warehouse w^* will never open.

5. A sequential decision-making approach

In this section, we outline a sequential decision-making model for comparing its solution with the one obtained by the integrated model proposed in Section 3. The traditional sequential decision-making approach for this problem follows a two-phase decision-making process. It typically makes the strategic location decision first followed by the operational inventory decision. In our problem framework, we first determine the number and locations of the warehouse to open, the warehouse-retailer assignments, and the sale price of the product for each open warehouse. Then, we determine the multi-echelon inventory replenishment policies for the open warehouses and the retailers assigned to them given the location and warehouse-retailer assignment decisions made in phase one. In the first phase, by ignoring the inventory-related cost components, we can formulate the problem as

$$S1: \max \sum_{w \in W} \sum_{S \subseteq I} R'_{w,S} x_{w,S}$$

$$\text{s.t.} \sum_{w \in W} \sum_{S \subseteq I: i \in S} x_{w,S} \leq 1, \ \forall\ i \in I,$$

$$x_{w,S} \in \{0,1\}, \quad \forall\ w \in W,\ S \subseteq I,$$

where $R'_{w,S} = \sum_{i \in S} r_i(p_w, d_{i,w})\lambda_i - F_w$ denotes the profit of serving retailers in *S* using warehouse *w* for $w \in W$ and $S \subseteq I$. The set-packing model (*S*1) is clearly a special case of (*P*), which can be solved similarly using branch-and-price.

In the second phase, we determine the multi-echelon inventory replenishment policies for the warehouses open and the retailers assigned to them in the network constructed by the phase one solution ($x_{w,S}$). Let $\mathbb{S}$ be the set of retailers served obtained by solving (*S*1) and *O* be the set of warehouses opened in the first phase. We can then formulate the second phase problem as follows:

$$S2: \min \sum_{w \in O} \sum_{S \in \mathbb{S}} I(w,S) \equiv \sum_{w \in O} \sum_{S \in \mathbb{S}} \min I(w,S).$$

We note that (*S*2) is a separable minimization problem as it can be decomposed for each pair of (*w*,*S*) which can be solved efficiently (cf. Roundy, 1985).

After (*S*1) and (*S*2) are solved, the total profit obtained from the sequential decision-making process equals to the objective value of (*S*1) minus the objective value of (*S*2). In the next section, we present the computational results and demonstrate the effectiveness of the integrated decision-making by comparing the solutions obtained from the integrated model and the sequential decision-making model.

6. Computational results

In this section, we summarize the computational results with the models outlined in the previous sections. All the instances were solved on a HP P4-2.8G workstation running the windows XP operating system. The reported computational times exclude input times. CPLEX 10.0 Solver is used to solve the test instances.

All the problem instances are randomly generated. For each retailer *i*, K_i, h_i, and ν_i are randomly generated in (0, 50] and λ_i is randomly generated in (0, 500]. For each warehouse *w*, $K_{w,0}$ is generated uniformly in $(\max_i K_i, 100]$; $h_{w,0}$ is generated uniformly in $(0, \min_i h_i)$; and F_w is generated uniformly in [50, 100]. The location of the warehouses and retailers are uniformly distributed over $[0, 2000] \times [0, 2000]$. Moreover, we assume that the per unit transportation cost is a stepwise function of the Euclidean distance and has the following structure:

$$c(d_{w,0}) = \begin{cases} 3 & \text{if } 0 < d_{w,0} \leq 100, \\ 6 & \text{if } 100 < d_{w,0} \leq 500, \\ 9 & \text{if } 500 < d_{w,0} \leq 1000, \\ 12 & \text{if } 1000 < d_{w,0} \leq 1500, \\ 15 & \text{if } 1500 < d_{w,0} \leq 2000, \end{cases}$$

$$g(d_{i,w}) = \begin{cases} 10 & \text{if } 0 < d_{i,w} \leq 100, \\ 15 & \text{if } 100 < d_{i,w} \leq 500, \\ 20 & \text{if } 500 < d_{i,w} \leq 1000, \\ 25 & \text{if } 1000 < d_{i,w} \leq 1500, \\ 30 & \text{if } 1500 < d_{i,w} \leq 2000. \end{cases}$$

The above transportation cost structures explicitly model the different quantity discount schemes. Practically, large volume shipments from the production site to the warehouses are delivered via the truckload transportation mode. In contrast, the relative small volume shipments from a warehouse to a retailer are usually made under the less-than-truckload mode. The per unit transportation cost under the less-than-truckload mode is higher than it under the truckload mode.

For each of the tested instances, we first solve the linear programming relaxation of the set-packing model (*P*) via the column generation approach. Then, we solve the pricing problem P_w for each *w* using the approach outlined in Section 4. We add the column with positive reduced cost to the master problem and start the next iteration. The initial set of columns contains all singletons. Tables 1 and 2 report the CPU time (in seconds) needed, the number of warehouses open, and the number of columns generated for different input sizes of the distribution network design instances ranging from three potential warehouse locations and 10 retailers to 25 potential warehouse locations and 100 retailers without implementing variable fixing and 10 potential warehouse locations and 30 retailers to 50 potential warehouse locations and 200 retailers with implementing variable fixing. We run each size of the instances 20 times and report the average values (rounded to the nearest integer). The columns titled "# W/H open", "# Retailers served", "# Columns", and "CPU time(s)" denote the number of warehouses open, the number of retailers served, the total number of columns added into the master problem during the solution process, and the average CPU time in seconds respectively. From Tables 1 and 2, we can see that variable fixing can help significantly reduce the solution time, e.g., for 25 locations and 100 retailers instance, the average CPU time taken by implementing variable fixing is only around 31% of that without implementing it. With the help of variable fixing, we are able to solve the problem with up to 50 potential warehouse locations and 200 retailers in 26 min in average.

We next fix the input size of the instances generated at 20 potential warehouse locations and 50 retailers, and compare our profit-maximizing model in which we can choose whether to serve each retailer (the set-packing model) with the traditional supply chain design model in which all the retailers should be served (the associated set-covering model). Tables 3 and 4 highlight the results of our computational study. In order to show how the inventory cost and the location cost affect the number and locations of the warehouses open and the number of retailers served, we use $\alpha I(w,S)$ and βF_w to substitute $I(w,S)$ and F_w in the original model, respectively, in which α is the inventory cost factor and β is the location cost factor. We use Z_I to denote the optimal objective value of the integrated profit-maximizing model and Z_A to denote the optimal objective value of the associated set-covering model in which all the retailers are served. The column titled "Δ (%)" denotes the profit improvement of implementing the integrated profit-maximizing model over the associated set-covering model in which all the retailer are served,

Table 1
Computational results without variable fixing.

Input		Output			
# W/H	# retailers	# W/H open	# retailers served	# columns	CPU time (s)
3	10	2	6	30	4.9
5	20	3	15	91	19.2
10	30	5	22	354	112.4
15	50	8	40	1210	368.4
20	80	8	64	4369	824.2
25	100	10	81	8535	1526.9

Table 2
Computational results with variable fixing.

Input		Output			
# W/H	# retailers	# W/H open	# retailers served	# columns	CPU time (s)
10	30	4	23	176	57.2
15	50	6	42	223	89.5
20	80	8	59	416	205.3
25	100	8	85	907	473.6
25	150	10	118	1135	615.9
50	200	16	142	2445	1536.3

Table 3
Computational results for varying inventory cost factor.

Input		Output			Comparison	
Inventory cost factor α	Location cost factor β	# W/H open	# retailers served	Objective value Z_I	Serve all Z_A	Δ (%)
1	1	8	38	159,762	147,240	8.5
3	1	7	36	154,702	137,512	12.5
5	1	7	36	140,737	121,517	15.8
10	1	5	35	131,847	111,357	18.4
15	1	4	34	121,922	100,573	21.3
20	1	2	34	109,021	88,204	23.6

Table 4
Computational results for varying location cost factor.

Input		Output			Comparison	
Inventory cost factor α	Location cost factor β	# W/H open	# retailers served	Objective value Z_I	Serve all Z_A	Δ (%)
1	5	8	38	150,562	145,502	3.4
1	10	8	37	148,432	137,503	7.9
1	30	7	36	140,035	126,011	11.1
1	50	6	36	128,501	114,398	12.3
1	80	4	35	115,503	102,384	12.8
1	100	4	35	103,011	89,892	14.5

and it is defined as

$$\Delta(\%) = \frac{Z_I - Z_A}{Z_A} \times 100.$$

From Tables 3 and 4, we observe that both the number of warehouses open and the number of retailers served are non-increasing with the increase of the inventory cost factor α and the location cost factor β, i.e., when either the inventory or location cost increases, both the number of warehouses open and the number of retailers served tend to decrease. We also observe that the profits obtained from the model with the flexibility to choose which set of retailers to serve are higher than those obtained from the covering model in which all the retailers must be served for all test instances. Furthermore, based on the randomly generated inputs, we can observe that the profit improvement is more significant when the inventory cost dominates the location cost in the system and the inventory cost tends to have more impact on the profit improvement.

In the rest of this section, we compare the solutions obtained from our integrated profit-maximizing model and the traditional sequential-decision making model. Table 5 summarizes the computational results obtained from solving the integrated profit-maximizing model and the non-integrated model with different input sizes. We use Z_S to denote the optimal profit obtained from the traditional sequential decision-making procedure, which equals to the optimal objective value of model ($S1$) minus the optimal objective value of model (S_2). The column titled "Δ' (%)" denotes the profit improvement of implementing the integrated profit-maximizing model over the traditional non-integrated approach and it is defined as

$$\Delta'(\%) = \frac{Z_I - Z_S}{Z_S} \times 100.$$

From Table 5, we can observe that the number of warehouses open and the number of retailers served by the integrated model

Table 5
Integrated vs. non-integrated model.

Input		Output of integrated model			Output of non-integrated model			Comparison
# W/H	# re	# W/H open	# re served	Objective value Z_I	# W/H open	# re served	Objective value Z_S	Δ' (%)
5	10	2	6	41,429	3	8	37,526	10.4
5	20	3	15	80,017	4	18	71,893	11.3
5	30	4	20	95,716	5	25	85,460	12.1
5	40	4	27	124,637	5	34	109,715	13.6
5	50	5	38	161,009	5	45	139,886	15.1
10	20	4	17	75,939	6	19	67,863	11.9
10	40	5	26	111,939	8	38	98,624	13.5
10	60	6	45	202,858	8	56	175,179	15.8
10	80	7	59	246,284	9	73	211,040	16.7
10	100	8	70	274,637	9	92	234,732	17.0
20	20	4	16	67,956	5	18	60,298	12.7
20	40	6	31	125,456	8	39	109,952	14.1
20	60	8	52	171,187	10	57	147,447	16.1
20	80	8	64	289,576	11	71	247,078	17.2
20	100	10	80	370,341	11	91	304,306	21.7

is consistently smaller than those obtained from the non-integrated model. The integrated model outperforms the sequential decision-making model by more than 10% in terms of the total profit. The profit improvement ranges from 10.4% to 21.7% and tends to increase when the input instance size becomes larger.

7. Extension and generalization

In this section, we describe some extensions and generalizations of our model by introducing an additional cost term $H(w,S)$ to capture more realistic situations, and show that the associated pricing problem can still be solved efficiently.

7.1. Business volume dependent locating and operating cost at the warehouse

In our model setting, we have assumed that there is only a fixed locating and operating cost F_w associated with warehouse w. In order to model a more general and realistic setting, we can employ a cost term $H(w,S)$ to incorporate costs that are dependent on the total annual business volume served by warehouse w. In this case, we may define

$$H(w,S) = g_w\left(\sum_{i\in S}\lambda_i\right),$$

where g_w is a warehouse-dependent cost function, which we may assume to be concave as in Shen (2006).

7.2. Routing cost

In our model setting, we have assumed that all the outbound logistics shipments are made directly from each warehouse open to each retailer served which is the same as the shipment pattern in the traditional uncapacitated facility location problem. By considering a more general and realistic situation and assuming each warehouse sends a vehicle to visit the retailers served at some fixed frequency, we may define a concave function $H(w,S)$ to capture the routing cost. Shen and Qi (2007) show that $H(w,S)$ is a function of the number of visits per year, the number of retailers served, the capacity of the vehicle employed, and the distance between warehouse w and each retailer i served. They use simulation to show that this concave function is a very good approximation to the routing cost.

7.3. Stochastic demand

When demand is stochastic and independent (e.g., we know that each retailer i faces a demand with mean λ_i and variance σ_i), both warehouses and retailers need to maintain a suitable level of safety stocks to meet the specified service level promised to customers. Let $L_{w,0}$ denote the replenishment lead time from the supplier to warehouse w. Then, $L_{w,0}\sum_{i\in S}\sigma_i^2$ is the total variance of lead time demand experienced by warehouse w. Let $\kappa_{w,0}$ be a controlling parameter which corresponds to the service level. Similarly, let $L_{w,i}$ denote the replenishment lead time from warehouse w to retailer i and κ_i be a controlling parameter that corresponds to the service level. With these notations, we can approximately measure the safety stock cost associated with warehouse w serving retailers set S by

$$h_{w,0}\kappa_{w,0}\sqrt{L_{w,0}\sum_{i\in S}\sigma_i^2}+\sum_{i\in S}h_i\kappa_i\sigma_i\sqrt{L_{w,i}},$$

where the first term denotes the safety stock cost incurred by warehouse w and the second term represents the safety stock cost incurred by retailers in S. We note that the second term is separable in i. In our model, we can merge this term into the transportation cost term. For ease of exposition, we can thus ignore this term. Thus, we can define the system safety stock costs due to demand uncertainty as

$$H(w,S) = h_{w,0}\kappa_{w,0}\sqrt{L_{w,0}\sum_{i\in S}\sigma_i^2}.$$

In the aforementioned three cases, the pricing problem becomes

$$\min_{S\subseteq I} I(w,S)+H(w,S)-\sum_{i\in S}a_i(p_w).$$

The pricing problem can be efficiently solved for any fixed p_w (cf. Romeijn et al., 2007). By allowing p_w to change, it is not difficult to see we can apply the similar idea developed in Section 4 to obtain the optimal solution.

8. Conclusions and future research

In this paper, we study a logistics network design problem with vendor managed inventory in which the company is in charge of managing inventory for its downstream warehouses and retailers, and can choose whether to fulfill each retailer's demand. We formulate the problem as a set-packing model and solve it using branch-and-price. The pricing problem that arises from each iteration of the column generation procedure is an interesting nonlinear IP problem. We propose an efficient algorithm which runs in $O(n^2 \log n)$ time to tackle it. Furthermore, we propose a heuristic to speed up the column generation procedure. We use extensive computational experiments to compare the solution of our integrated decision-making model with the one of the traditional sequential decision-making model. The average benefit ranges from 10.4% to 21.7% in terms of the total profit. The computational results also shed insights on the benefits of our model with the supplier having retailer-serving flexibility over the traditional model that requires all the demands should be served.

In the literature, most of the integrated supply chain network design models assume one level of warehouses. Practically, we might have two or even more level of facilities. Also currently, we can only deal with medium size problem instances. Therefore, we want to explore the possibility of designing efficient approximation algorithms for it and its variants (cf. Du et al., 2010; Du et al., this issue; Xu and Du, 2006; Xu and Yang, 2009; Xu and Zhang, 2008; Zhang, 2006), and being able to solve large-scale instances of the problem. Each of these could be an interesting topic for further investigation in future research.

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